Simultaneous Heat and Mass Transfer from Multicomponent Condensing Vapor-Gas Systems

Multicomponent vapor condensation in the presence of noncondensing gas has been analyzed using numerical solutions to a set of linearized rate equations that describe the simultaneous heat and mass transfer to the condensate film. For specific examples of methanol and water and ethanol and toluene condensing from turbulent air streams in vertical tube and on horizontal tube condensers, it is shown that the alcohol concentrations in both the liquid and gas phases increase as condensation from initially superheated, alcohol-rich gas phases progresses; the liquid phases also become richer in alcohol. The mass fluxes of the vapors vary with their respective mass transfer coefficients and local concentration driving forces. In the vertical tube condensers, the flux of the alcohols go through maximums. Gas phase saturation is approached as the mixtures cool and the final traces of vapor become more difficult to condense. Interface properties are computed from limited *VL* equilibrium data, heat of solution data, and a predictive equation. The techniques of solution and analysis have general application to the design of cooler-condensers in either vertical or horizontal configurations.

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SCOPE

In 1929 and 1930 Othmer and Colburn and Hougen published the first comprehensive results on vapor condensation in the presence of a noncondensable gas. From their work and from others that followed, the basic principles of the process were established: vapor and sensible heat are transported simultaneously from the bulk gas mixture to the condensate film, the interface of which prevails in an equilibrium state (Colburn and Hougen, 1934); the diffusing vapor carries sensible heat to the interface (Ackermann, 1937) where it is released along with the latent heat of condensation; in certain cases the gas phase can be expected to superheat or subcool (Colburn and Edison, 1941) and this can be predicted by an established test (Bras, 1953) and in a metastable subcooled state, a potential for fog formation exists (Steinmeyer, 1972). All of these basic concepts were incorporated into analytical solutions for predicting pure vapor-gas behavior in cooler-condensers (Schrodt, 1972).

The objective of the present study is the development of a model of the multi vapor-gas cooler-condenser problem in terms of a set of linear differential equations which, when supplemented with equations describing the equilibrium state of the interface, can be numerically solved for both horizontally or vertically aligned condensers. These solutions give a set of property profiles for the gas, condensate, and coolant phases as a function of area. These can then be used to analyze the dynamics of the process.

CONCLUSIONS AND SIGNIFICANCE

Multicomponent vapor condensation in the presence of noncondensing gas has been analyzed by considering simultaneous heat and mass transfer to a concentration and temperature varying gas-liquid interface. A general model of the process has been developed and numerically solved for specific cases of methanol and water and ethanol and toluene condensing in the presence of air in turbulent flow in vertical and horizontal tube condensers. For the methanol-water system the first traces of condensate were very rich in the less volatile component, water, but as this was progressively removed, the condensate became richer in the more volatile component, methanol, as shown in Figure 5 in the text. For the ethanol-toluene system the first traces of condensate were rich in the more volatile component, ethanol. Mass fluxes varied in accordance with their respective mass transfer coefficients and local concentration driving forces; preferential condensation prevailed. Core stream saturation was approached, indicating in a relative way that the sensible heat transfer was greater than the rate of mass transfer. This effect should not be construed as general because other systems may show a reverse tendency. It is also shown that the varying equilibrium state that prevails along the interface must be predicted with reasonable accuracy in terms of both temperature and composition in order to predict the local heat and individual mass transfer rates.

The proposed model of the process and its numerical solution make it possible to design and analyze the performance of many cooler-condensers.

Pure vapor condensation in the presence of noncondensable gases has been the subject of much research since the early studies of Colburn and Hougen (1930) and Othmer (1929), and many cooler condenser design procedures have been published (Colburn and Hougen, 1934; Mizushina et al., 1960; Bras, 1957; Algermessen, 1958; Coughanowr and Stensholt, 1964; Ivanov, 1962; Hoffman, 1971; Schrodt, 1972).

Very little work can be found regarding multi vapor-gas systems. Porter and Jeffreys (1963) condensed ethanol and water from air and nitrogen on a horizontal single tube condenser and presented a graphical design procedure for binary vapor-gas systems. Mizushina et al. (1964) proposed a simplified three-point method of design based upon inlet, outlet, and center point conditions in a condenser. Based upon their experimental results in a single tube vertical condenser, the method predicted the area within ± 20%. Schrodt and Gerhard (1968) obtained experimental data on the condensation of methanol and water from an air stream in turbulent flow across water cooled, vertical cylinders. Their results confirmed the existence of equilibrium between the adjacent gas and liquid phases at the interface.

The objective of this study is the development of a model of the multi vapor-gas cooler condenser problem in terms of a set of linear differential equations. These equations, when supplemented with those describing the equilibrium state of the interface, can be numerically solved for either horizontally or vertically aligned condensers, resulting in a set of property profiles for the gas, condensate, and coolant phases as a function of area. Solutions are obtained for a methanol-water-air system and an ethanol-toluene-air system condensing in single tube, vertical condensers, and the same ethanol-toluene-air system condensing on the outside of a horizontal tube in shell condenser. Plots of temperatures, heat, and mass transfer rates and heat loads are presented as a function of the condenser areas and then discussed in terms of the preferential nature of the molecular transport, the approach to core stream saturation and the dynamics of the overall process.

MATHEMATICAL ANALYSIS

Consider an unsaturated multivapor noncondensable gas mixture flowing in either a vertically or horizontally aligned condenser with a countercurrent flow of coolant. The coolant's temperature is everywhere low enough to provide a surface below the saturation temperature of the mixture, a condition which develops concentration and temperature gradients essential for the transfer of both heat and mass. It is also prescribed that the condensed vapor components form a smooth, homogeneous film whose composition varies along the surface in the overall direction of gas flow but does not vary in the direction normal to the gas flow. For vertical tube condensers the liquid composition at each cross section is a direct function of the integrated mass fluxes evaluated from the inlet to the cross section in question, and for horizontal condensers it is a function of the local mass fluxes streaming to the interface at that point.

Over a differential area of the condenser, the core stream sensible heat and mass transfer rates are given, respectively, by

$$GC_p \frac{dT_g}{dA} = -h_g \frac{\epsilon}{e^{\epsilon}(1-e^{-\epsilon})} (T_g - T_s) \qquad (1)$$

$$\frac{dV_j}{dA} = -k_{gj}(P_{gj} - P_{sj}) \quad j = 1, 2 \ldots m \qquad (2)$$

$$\frac{dV_n}{dA} = 0 \qquad (3)$$

The Ackermann (1937) correction factor $\epsilon/(1-e^{-\epsilon})$ appearing in Equation (1) accounts for sensible heat transferred by diffusing vapors. In the absence of condensation this factor equals 1.0 but increases positively as the rate of condensation increases. The combined factor $\epsilon/e^{\epsilon}(1-e^{-\epsilon})$, or as it is sometimes expressed $\epsilon/(e^{\epsilon}-1)$, accounts for the affect of mass transfer on the temperature of the gas mixture. For multivapor condensation $\epsilon = \Sigma N_j C_{pj}/h_g$. As ϵ increases the combined factor predicts the temperature of the mixture increases (Colburn and Drew, 1937). Equation (2) is applied to each of the condensing species. The form of the equation results from an extension of the film theory of mass transfer, which is usually applied to binary mass transfer problems, to multicomponent mass transfer. This assumes that each component diffuses in the presence of the nondiffusing, noncondensable gas as if there were no interaction with the other diffusing vapors. When the diffusivities are all equal this is virtually true. Although there are more exact methods for evaluating the individual mass transfer rates when the diffusivities are not equal, they complicate the mathematics of this problem to the extent that the solution is extremely tedious even with the aid of a computer. Therefore, in this work the film theory of mass transfer has been used with binary mass transfer coefficients.

The temperature of the coolant increases as sensible and latent heats are conducted through the condensed film, tube wall, and a fictitious coolant film. For countercurrent flow this rate is expressed as

$$wc_w \frac{dT_w}{dA} = -h_o' (T_s - T_w) \qquad (4)$$

For concurrent flow conditions, this equation holds if the right-hand side is made positive. The coefficient h_o' is a combined heat transfer coefficient based upon the resistances of the condensed film, tube wall, any scale thereon, and a fictitious coolant film. While all of the latter are nearly constant in any condenser, the resistance of the condensate film varies with thickness and its physical properties, including composition. By itself this variation appears significant, particularly for vertical condensers where the thickness changes from zero at the top to a maximum at the bottom; however, when combined with the other resistances and compared with those in the gas phase, it is not.

The state of the gas-liquid interface in a cooler-condenser cannot be determined directly; however, previous experimental investigations by Schrodt and Gerhard (1965, 1968) with single vapor-gas and binary vapor-gas systems have verified that distinct equilibrium states prevail along the interface. The gas-condensate, interface partial pressures P_{sj} are therefore the equilibrium partial pressures corresponding to the interface temperature T_s, compositions x_j, and activity coefficients γ_j:

$$P_{sj} = \gamma_j x_j P_j^0 \qquad (5)$$

where

$$P_j^0 = \phi_j (T_s) \qquad (6)$$

$$\gamma_j = \psi_j (T_s, x_j) \qquad (7)$$

If the liquid interface compositions x_j, temperature T_s, and activity coefficients γ_j can be calculated either explicitly or implicitly using a limited amount of VL equilibrium data and a predictive equation, then the gas interface concentrations y_j and partial pressures P_{sj} can be obtained through Equation (5). This procedure has been successfully employed by Schrodt and Gerhard (1968).

If Equation (2) is multiplied by λ_j and combined with Equations (1) and (4), a heat balance can be obtained at the interface. Hence,

$$h_o' (T_s - T_w) = h_g \frac{\epsilon}{1 - e^{-\epsilon}} (T_g - T_s) + \Sigma k_{gj} \lambda_j (P_{gj} - P_{sj}) \qquad (8)$$

This equation can be rearranged to give

$$T_s = \frac{h_g \dfrac{\epsilon}{1 - e^{-\epsilon}} T_g + \Sigma k_{gj}\lambda_j (P_{gj} - P_{sj}) + h_o' T_w}{h_o' + h_g \dfrac{\epsilon}{1 - e^{-\epsilon}}} \qquad (9)$$

While this equation appears explicit in T_s, it must be remembered the P_{sj} terms are nonlinear functions of T_s. It follows that

$$wc_w \frac{dT_w}{dA} = e^{\epsilon} G C_p \frac{dT_g}{dA} + \Sigma \lambda_j \frac{dV_j}{dA} \qquad (10)$$

This equation may be integrated directly and solved for T_w subject to the b.c.

$$T_w = T_{wo}$$

$$T_g = T_{gi} \quad @A = 0$$

$$V_j = V_{ji}$$

giving

$$T_w = T_{wo} + \frac{e^{\epsilon} G C_p}{wc_w} (T_g - T_{gi}) + \frac{\Sigma \lambda_j (V_j - V_{ji})}{wc_w} \qquad (11)$$

This equation may be substituted into Equation (9) to eliminate the variable T_w.

The local rate of heat transfer in the condenser is given by

$$q = - wc_w \frac{dT_w}{dA} \qquad (12)$$

and the heat load up to a given A by

$$Q = - wc_w \int_0^A \frac{dT_w}{dA} dA \qquad (13)$$

Equations (1) to (13) define the simultaneous heat and mass transfer processes that take place in cooler condensers void of any core stream condensation process. Heat transfer from radiation effects, condensate subcooling, and condensate mixing have not been included in the analysis because these are usually negligible. The exact analytical solution to the above set of equations has been obtained for the case of $j = 1$ and constant coefficients (Schrodt, 1972). To obtain a solution to the problem when $j > 2$, the set of $j + 1$ linear differential equations, Equations (1) and (2), must be solved numerically, simultaneously with an interative solution to Equations (5) to (7), and (9) and the equations

$$P_{gj} = \frac{V_j P_t}{\Sigma V_j} \quad j = 1, 2 \ldots n \qquad (14)$$

$$x_j = (V_{ji} - V_j)/\Sigma (V_{ji} - V_j) \quad j = 1, 2 \ldots m \qquad (15)$$

$$A > 0, \text{ vertical}$$

$$x_j = \frac{k_{gj}(P_{gj} - P_{sj})}{\Sigma k_{gj}(P_{gj} - P_{sj})} \quad j = 1, 2 \ldots m$$

$$A = 0, \text{ vertical}$$

$$A \geqq 0, \text{ horizontal} \qquad (16)$$

The geometric constraints assigned to Equations (14) to (16) result from the following assumptions: (1) There are no liquid phase concentration gradients in the direction of transport, that is, a homogeneous liquid solution prevails at each locus of the condenser, (2) in a horizontal condenser and at the inlet of a vertical condenser, the concentration of the liquid phase is that resulting from the immediate mass transport to the locus, and (3) at all other loci in a vertical condenser, the condensate is the bulk average that has accumulated at that location. In the ternary vaporization studies carried out by Toor and Sebulsky (1961) in a wetted wall column, the assumption of negligible diffusive resistance in the liquid phase was attributed to bulk mixing and then confirmed by experimental results. Colburn and Drew (1937) in their theoretical studies on the condensation of methanol and water in a vertical condenser, and Schrodt and Gerhard (1968) in their experimental work on the condensation of the same vapors but in the presence of air predicted that the composition of the first trace of condensate was the same as that of the vapor streaming to the bare, cold surface. Thus, assumption (2) is supported by previous results.

Equations (1) and (2) comprise a set of $j + 1$ first-order linear differential equations representing the process. The terms T_g, V_j, and T_s are the dependent variables, and A is the independent variable. P_{gj} and P_{sj} are algebraically dependent for their values through Equations (5) to (7) and (14) to (16). All other terms must be taken as constants having values suitably averaged either over the whole condenser or over shorter intervals.

SOLUTION TECHNIQUE FOR $m = 2$

For binary vapor-gas condenser problems $m = 2$, there are three differential equations containing eight variables, T_g, T_s, V_1, V_2, P_{g1}, P_{g2}, P_{s1} and P_{s2}. The variables T_g, V_1, and V_2 are retained for solution by stepwise integration, and P_{g1} and P_{g2} are directly eliminated by Equation (14). Variables T_s, P_{s1}, and P_{s2} are implicitly evaluated using Equations (5) to (7), (9), (11), (15), and (16), and the stepwise integrated values of T_g, V_1, and V_2. For a vertical condenser, this entails a double iterative procedure at $A = 0$ and a single iterative procedure for $A > 0$. At $A = 0$, T_s is assumed equal to $T_{wo} + \Delta$, and x_1 is assumed equal to 0.5. These values are then applied to Equations (7), (6), and (5), in that order, to evaluate P_{s1} and P_{s2}. Equation (16), which for $m = 2$ can be solved in quadratic form for x_1, can then be used to check the assumed value of x_1. If it is incorrect additional iterations must be performed until Equation (16) is satisfied. Next, aT_s is calculated using Equation (9), and this is compared with the assumed value. If T_s is incorrect the entire procedure is repeated until both x_1 and T_s are validated at $A = 0$. The first stepwise integration of Equations (1) and (2) can then be performed and the results applied to Equation (11) to calculate T_w. For $A > 0$, x_1 is calculated via Equation (15). With x_1 and an assumed value of $T_s = T_w + \Delta$, Equations (5) to (7) are used to calculate P_{s1} and P_{s2} and then T_s is checked using Equation (9). Additional iterations are usually necessary to converge on the correct T_s. Finally, the differential equations are solved in the second step.

For a horizontal condenser, the double iterative procedure applied to the vertical condenser at $A = 0$ is applied at all $A \geq 0$.

The above procedures have been computer programmed using a fourth-order Runge Kutta method of stepwise integration for single tube vertical and single tube horizontal condensers and $m = 2$, and are available *as a supplement*[*] to this paper.

APPLICATIONS

Because different chemical systems behave differently in the same condenser and any chemical system behaves differently in different condensers it was impossible to generalize solutions into a single dimensionless form. To illustrate this fully, three cases involving two systems and the two geometries were examined. The two systems were methanol-water-air and ethanol-toluene-air. The former is an example of a more volatile component, methanol, having a smaller molecular diffusivity than its accompanying vapor; the ethanol in the other system has both a greater volatility and diffusivity than the toluene. Also, the ethanol-toluene system forms an azeotrope.

Cases I and II

The computer program was used to design 2.54 cm. I.D. vertical, single tube condensers for condensing methanol and water (Case I) and ethanol and toluene (Case II) from their mixtures with air. The mixtures entered their respective condensers at a prescribed temperature, pressure, composition, and flow rate, and condensation progressed until a specified temperature or core stream saturation was reached. Cooling water entered the annuli at the bottom at a known flow rate and the outlet temperature was specified. Unknown terminal temperatures and concentrations were estimated from overall material and enthalpy balances that required guesses of the gas phase exit compositions. Readjustments of these estimates were necessary after initial computations.

By using averaged properties, partial pressures and flow rates, gas-phase heat and mass transfer coefficients were calculated from the following correlation (Treybal, 1968):

$$j = h_g/GC_p\,Pr^{2/3} = \frac{k_g\,P_{BM}\,M_{av}}{G}\,Sc^{2/3} = 0.023\,Re^{-0.17} \tag{17}$$

Case III

The computer program was also used to design a single tube in shell horizontal condenser for condensing ethanol and toluene from their mixture with air. This hypothetical condenser had a single tube, 1.27 cm. O.D., centered and baffled every 7.62 cm. in a 5.08 cm. I.D. shell. Cooling water flowed through the tube concurrent to the shell side gas flow. Terminal conditions were either specified or estimated as in cases I and II. Heat and mass transfer coefficients were calculated from the following correlation obtained by Schrodt and Gerhard (1968):

$$j = (0.75)\,0.79\,Re^{-0.475} \tag{18}$$

The constant (0.75) was applied to this equation to account for the reduced turbulence associated with flow across the single tube.

Applications of the j-factor analogy and their correlations to problems involving simultaneous heat and mass transfer have been validated for both parallel (Stern and Votta, 1968) and cross flow geometries (Schrodt and Gerhard, 1965) for single vapor-gas systems. The extension of the correlations to multicomponent diffusing systems, particularly where the component's diffusivities are not greatly different appears justifiable for design procedures where averaged coefficients are used. At 25°C the diffusivities of toluene, ethanol, methanol and water in air are 0.084, 0.121, 0.160, and 0.256, respectively. The combined coefficient h_o' was taken as a constant 1700 watts/m^2°C for all cases. Methanol-water isothermal VL equilibrium data at 25°C (Butler et al., 1933), and heats of solution at infinite dilution (Benjamin, 1959) and ethanol-toluene VL data at 35 to 85°C (Hala, 1968) were used with the Van Laar equations to predict all the interface equilibrium states. To assist in analyzing results and assessing the potential for core stream condensation, data for several x-y-t equilibrium diagrams were also computed. The P_{BM} in Equation (17) was taken as the logarithmic mean of the air.

RESULTS AND DISCUSSION

Assigned and computed terminal values of T_g, P_{g1}, P_{g2}, q and so forth and average values of several other quantities, h_g, P_{BM}, $\epsilon/1 - e^{-\epsilon}$ and so forth are given in Table 1. Numbers in parentheses correspond to the preliminary estimated values. Changes in coolant, interface and mixture temperatures, heat and mass fluxes, and heat loads with condenser length are given in Figures 1 to 3 for Case I. Changes in the mass fluxes with condenser length are given in Figure 4 for Cases II and III. In Figures 5 and 6 isothermal x-y equilibrium curves of the binary VL systems have been plotted in terms of the more volatile components, methanol and ethanol. Also, in these figures may be seen curves representing changes in the gas phase mixture properties $\dfrac{\sum^{m=2} P_{gj}}{P_T}$ vs. y_1 and corresponding condensate interface properties $\dfrac{\sum^{m=2} P_{sj}}{P_T}$ vs. x_1. The latter are located below and to the left of the former. It may be recalled, the interface properties correspond to equilibrium states and therefore their curves intersect the x_1 equilibrium curves at the same temperatures.

The mixtures entered their respective condensers at $T_g = 71.1$°C and $\dfrac{\sum^{m=2} P_{gj}}{P_T} = 0.35$ corresponding to superheated states as shown in Figures 5 and 6. Had the methanol-water-air mixture entered at the same mole ratio of methanol vapor $y_1 = 0.61$ but with vapor partial pressures of $\dfrac{\sum P_{gj}}{P_T} \geq 0.68$, it would have been saturated or subcooled and a potential for core stream condensation would have existed immediately. An in situ fog would have ensued with $x_1 = 0.251$. As the mixtures passed through the condensers, vapor and sensible heat were transported to the cooled surfaces forming binary liquid films at rates equal to the respective products of the transfer coefficients and driving forces. As this occurred, the partial pressures of the vapor species and the temperatures of the gas, coolant, and condensate decreased as indicated in Figures 1, 5, and 6.

For Case I the mass flux of water to the surface at the

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[*] The supplement has been deposited as Document No. 02093 with the National Auxiliary Publications Service (NAPS), c/o Microfiche Publications, 305 East 46 St., N.Y., N.Y. 10017 and may be obtained for $1.50 for microfiche and $5.00 for photo copies.

Table 1. Terminal and Averaged Property Values

Methanol-Water-Air

L, m	0	2.12
V_1, kg mol/s	0.392×10^{-4}	0.221×10^{-4}
V_2, kg mol/s	0.252×10^{-4}	0.0625×10^{-4}
V_3, kg mol/s	1.197×10^{-4}	1.197×10^{-4}
T_g, °C	71.1	37.7
T_s, °C	42.9	29.5
T_w, °C	35.0	26.3
P_{g1}, N/m^2	2.158×10^4	1.515×10^4
P_{g2}, N/m^2	1.388×10^4	0.0427×10^4
P_{s1}, N/m^2	1.671×10^4	0.1105×10^4
P_{s2}, M/m^2	0.619×10^4	0.0248×10^4
q, W/m^2	3507	1412
Q, J/s	0	1594
P_{BM}, N/m^2	$0.79(0.8) \times 10^5$	
h_g, W/m^2 °C	55.19	
$\epsilon/(1 - e^{-\epsilon})$	1.183(1.2)	
k_{g1}, kg mol/sm^2 (N/m^2)	1.93×10^{-8}	
k_{g2}, kg mol/sm^2 (N/m^2)	2.60×10^{-8}	
P_T, N/m^2	1.013×10^5	

Ethanol-Toluene-Air

	Vertical		Horizontal	
L	0	1.81	0	2.42
V_1	0.392×10^{-4}	0.190×10^{-4}	0.243×10^{-4}	0.118×10^{-4}
V_2	0.252×10^{-4}	0.113×10^{-4}	0.158×10^{-4}	0.0719×10^{-4}
V_3	1.197×10^{-4}	1.197×10^{-4}	0.743×10^{-4}	0.743×10^{-4}
T_g	71.1	40.6	71.1	39.9
T_s	42.6	31.1	39.8	30.1
T_w	35.0	26.5	32.2	26.2
P_{g1}	2.158×10^4	1.283×10^4	2.158×10^4	1.281×10^4
P_{g2}	1.388×10^4	0.763×10^4	1.388×10^4	0.781×10^4
P_{s1}	1.498×10^4	0.810×10^4	1.309×10^4	0.774×10^4
P_{s2}	0.743×10^4	0.425×10^4	0.644×10^4	0.400×10^4
q	3387	2012	1689	876
Q	0	1562	0	1035
P_{BM}	$0.78(0.8) \times 10^5$		$0.79(0.8) \times 10^5$	
h_g	74.27		57.35	
$\epsilon/(1 - e^{-\epsilon})$	1.24 (1.25)		1.19 (1.20)	
k_{g1}	2.26×10^{-8}		1.82×10^{-8}	
k_{g2}	1.94×10^{-8}		1.56×10^{-8}	
P_T	1.013×10^5		1.013×10^5	

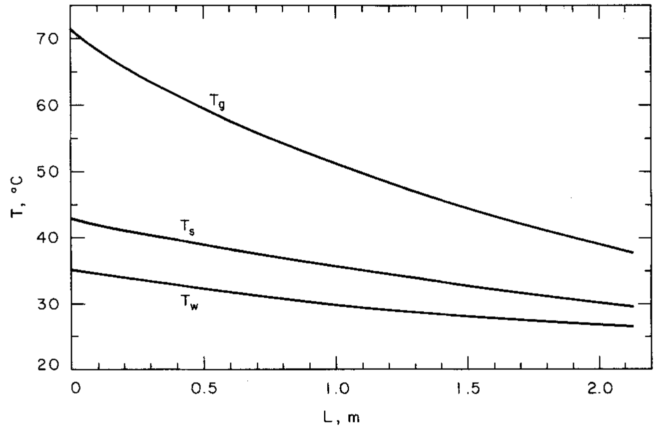

Fig. 1. Computed temperatures as a function of condenser length for methanol-water-air system.

inlet point was much greater than that of the methanol. This is attributed to the low relative volatility of water and its high diffusivity. As condensation progressed, the gas mixture became depleted of water, and the flux of water decreased correspondingly, while the flux of methanol increased slightly, reached a maximum, and then decreased. This is shown in Figure 3. For this situation the first traces of condensate were very rich in water, but further down the condenser the condensate became increasingly richer in methanol. The composition of the methanol in the gas mixture y_1 increased because there was always a smaller fraction of it being removed from the gas mixture. At the exit to the condenser, at $L = 2.12$ m, the gas mixture became saturated at 37.7°C and any further cooling would have resulted in a metastable subcooled state having a potential for fogging, particularly if any nuclei were present. The dashed lines in Figure 5 connect the adjacent core gas phase and interface properties; they are not representative of property profiles along the paths of transport but do show that great composition differences existed between adjacent gas and liquid phases that were not at equilibrium.

Curves in Figure 2 show the local heat transfer rate and heat load as a function of condenser length. As expected, the rate decreased rapidly as the mixture passed through the condenser, indicating that condensation of the final traces of the vapors became increasingly difficult as the concentration of noncondensable gas increased, and the mass transfer driving forces decreased. This is

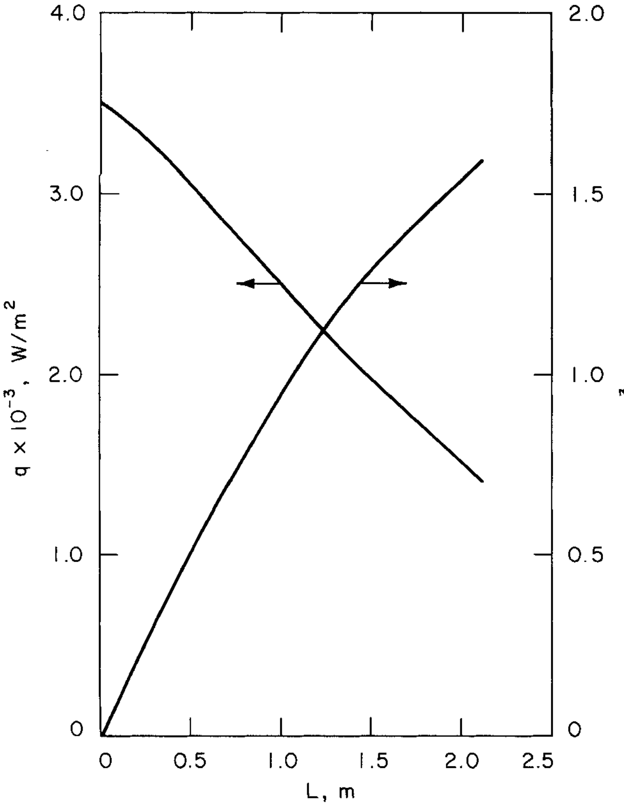

Fig. 2. Heat transfer fluxes and rates expressed as a function of condenser length.

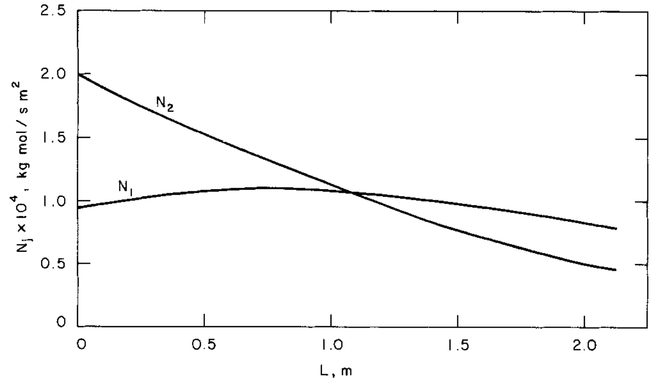

Fig. 3. Molar fluxes of methanol (1) and water (2) as a function of condenser length.

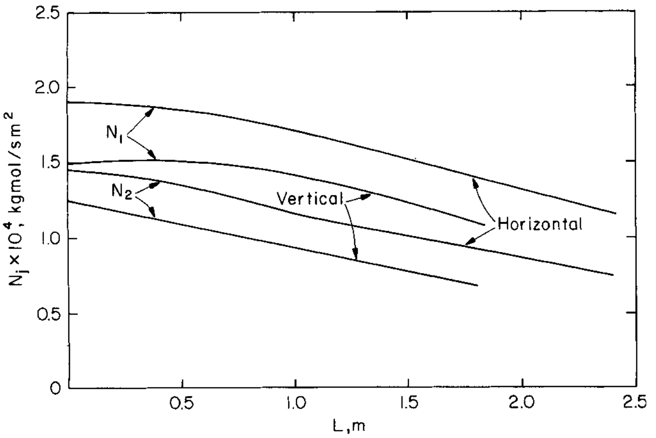

Fig. 4. Molar fluxes of ethanol (1) and toluene (2) as a function of condenser length.

further exemplified by the decreasing rate of the heat load.

When the ethanol and toluene were condensed from their mixture with air in the hypothetical, vertical tube condenser (Case II), the condensate formed was rich in the more volatile, ethanol, and continued to become slightly richer as condensation progressed. This is opposite to Case I for which the condensate was rich in the less volatile water. The converse effect is the result of the diffusivity of ethanol being greater than that of toluene, and the relative volatility of ethanol in toluene being only 1.9; in Case I, the water had a greater diffusivity, and the relative volatility of methanol in water was 2.7 at the inlet to the condenser. It is apparent that relative diffusivities, volatilities, and gas mixture concentrations of the vapors are all influential in directing the course of the condensation process, and hence, the composition of the condensate formed and the residual vapors. As shown in Figure 6, there was not a great change in or difference between the composition of the condensate and the vapors

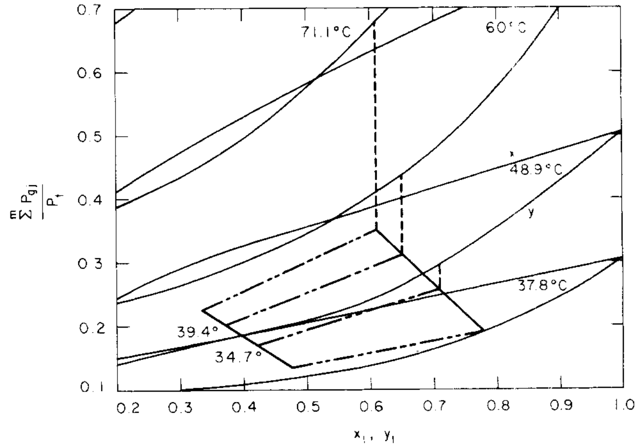

Fig. 5. Computed gas phase and interface properties relative to the x-y equilibrium curves of methanol (1) and water (2).

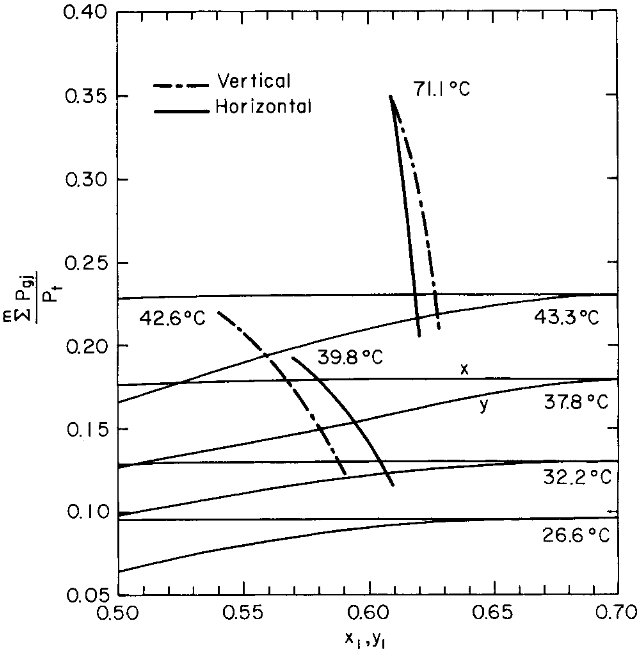

Fig. 6. Computed gas phase and interface properties relative to the x-y equilibrium curves of ethanol (1) and toluene (2).

along the length of the condenser. The individual fluxes of ethanol and toluene which are plotted as a function of the length of the condenser in Figure 4 indicate the rate of ethanol condensation reached a maximum at $L = 0.36$ m. and remained greater than the toluene flux. This maximizing effect, which was also predicted for the methanol in Case I, is not fully understood; however, it is clearly related to the vertical geometry of the condenser and therefore the local composition of the condensate. When this system was condensed in the horizontal condenser, no maximum was predicted because in this case the ethanol composition was always a direct function of the vapors streaming to the local surface. Dissimilarities between the results for Cases II and III are a consequence of many factors: principally, differences in transfer coefficients and the mechanics of the condensate flow which influenced the local condensate composition.

It is apparent from the predicted results for these two binary-vapor-air systems and the two simple geometries that the condensation of multicomponent vapors from noncondensable gases is a very complicated process and requires a great deal of experimental verification. The foregoing mathematical analysis of the multicomponent vapor-gas cooler-condenser problem in terms of linearized rate equations, and their numerical solution for cases of binary vapor-gas systems condensing in horizontal and vertical condensers have been shown to be useful for designing condensers and analyzing their performance. The procedures predict local heat and mass transfer rates, heat loads, and gas, condensate and coolant properties as a function of condenser area.

NOTATION

A = condenser area, m^2
C_p = heat capacity of mixture, J/kg mol °C
c_w = heat capacity of coolant, J/kg °C
G = mixture flow rate, kg mol/s
h_g = gas-phase heat transfer coefficient, W/m^2 °C
h_o' = combined heat transfer coefficient, W/m^2 °C
j = factor
k_g = mass transfer coefficient, kg mol/sm^2 (N/m^2)
L = condenser length, m
M = molecular weight, kg/kg mol
N = mass flux, kg mol/sm^2
P = pressure, N/m^2
Q = thermal load, J/s
q = heat transfer rate, W/m^2
T = temperature, °C
V = gas flow rate, kg mol/s
w = coolant flow, kg mol/s
x = liquid mole fraction
y = vapor mole fraction
γ = liquid phase activity coefficient
λ = latent heat of condensation, J/kg mol
Pr = Prandtl number
Re = Reynolds number
Sc = Schmidt number

Subscripts

BM = logarithmic mean of noncondensable gas
g = gas
i = inlet
j = component index
o = outlet
s = interface
t = total
w = coolant
1 = methanol or ethanol
2 = water or toluene
3 = air

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